

## THE SPEED CONTROL OF THE INDUCTION MOTORS BY THE CHANGE OF SUPPLY VOLTAGE

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### Abstract:

The main point of this topic is a speed control method of induction motors by a change of the supply voltage, but the frequency of supply voltage is constant. In fact this control method is a regulation by the slip, but this is caused by change of the voltage. This method is useful for the special purposes. Because of this, the conditions of motor's stability are determined precisely.

### 1. Introduction

The basic principle of this control method is really simple. But it is not possible to use this method everywhere. For this reason we suppose that the load torque has the parabolic shape. The load curve and a torque characteristic of the motor have an intersection. The skid of the IM increases with decrease of the voltage. Thus, we have to use the motor with greater power than is the nominal. For this reason, this control method is suitable for small power range applications such as the ventilators, fans in air-conditions devices and so on.

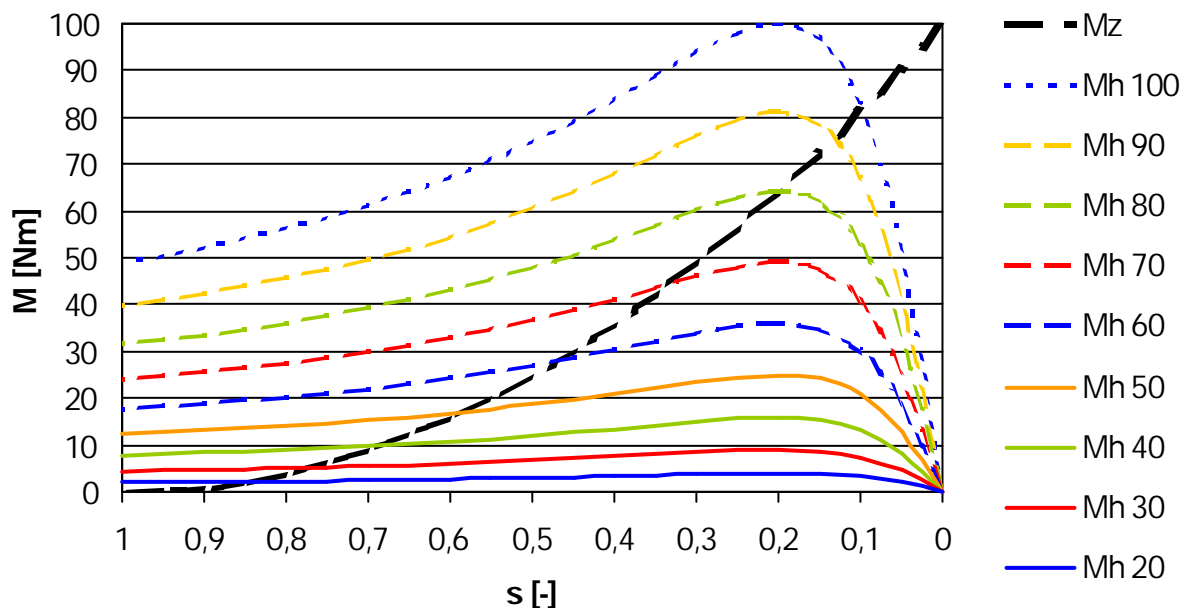


Figure 1. Torque characteristics of IMs – variable stator voltage and quadratic load characteristic. Intersection point means constant slipping of the shaft. (Mh 20 - number is a ratio of the stator voltage and nominal voltage)

## 2. The torque characteristic of IM:

It is possible to describe the torque of IM by Kloss equation. From the figure no. 2 it is clear that the deviation between the real values and the computed values is too high. The result is the same if we used the complicated version of Kloss equation no. 2. It is caused by the dependence between the rotor resistance and the rotor frequency. During a start or at low speeds a description of the IM torque characteristic has to respect a skin-effect (as opposed to “Kloss-form”). For exact description of the measured characteristic we propose an equation that respects a non-linearity of the motor. The differences between experiment and the equation (dashed line) are then small enough.

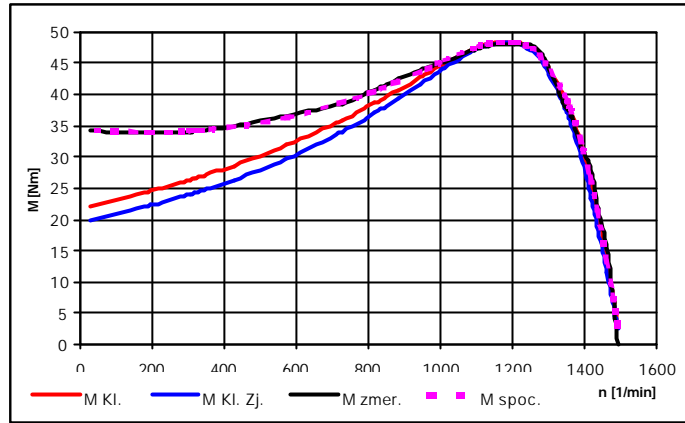


Figure 2. – IM torque characteristic and its approximations

*M Kl.zj.* – simplified Kloss equation

*M Kl.* – Kloss equation

*M zmer.* – measured characteristic

*M spoc.* – empiric form (almost as measured)

The inner torque of IM is defined by following equation,

$$M = \frac{3}{\omega_{1m}} \frac{R'_2}{s} \frac{U_1^2}{\left( R_1 + C_1 \frac{R'_2}{s} \right)^2 + X_{1\delta}^2} \quad (1)$$

Where  $C_1 \approx 1$  is a complex factor of a leakage – it will be neglected. If we compare the equation no.1 and the pullout torque, we obtain the Kloss equation no. 2.

$$\frac{M_i}{M_{i\max}} = \frac{2 * \left( 1 + \frac{R_1}{R'_2} * s_{zv} \right)}{\frac{s}{s_{zv}} + \frac{s_{zv}}{s} + 2 * \frac{R_1}{R'_2} * s_{zv}} \quad (2)$$

From this equation is derived the Kloss equation (3). This equation supposes that the rotor resistance is zero, but this condition isn't acceptable for small machines.

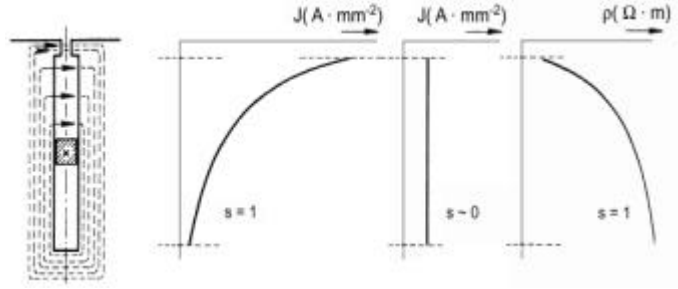
$$\frac{M_i}{M_{i\max}} = \frac{2}{\frac{s}{s_{zv}} + \frac{s_{zv}}{s}} \quad (3)$$

## 3. Deriving of accurate relations:

For exact torque description it is necessary to know diameters of stator coils. With respect to its resistance the “Kloss-form” would be very complicated and it would describe only one IM. That is why we decided to form empirical description of the rotor resistance as a function of a dc stator resistance and of the other experimental variable. An experimental variable respects different types of the rotor. It assumes that stator resistance is equal to the rotor resistance.

The skin effect causes that the current density decreases nearly exponential (see picture no.3).

$$J_2''(x) = J_2' * e^{-A * s * \frac{z}{l}} \quad (4)$$



Where  $J_2''(x)$  - current density in a slot,  
 $J_2'$  -DC ; current density (it is constant in a whole slot),

Figure 3. – Rotor current density as function of frequency

$A$  - constant (it is depend on type on the machine  $\approx 1.3 - 3$ ),  
 $z, l$  - position in the slot and the slot height.

Let's try to describe the dependence between the resistance  $\rho$  and position in the slot  $z$ . The current distribution must be the same as in the case with skin effect - in case that DC current flows through the conductor. Providing that the voltage has an equipotential shape and the conductor width is constant the following equation is valid

$$\rho(z) * J(z) = const \quad (5), \text{ from this equation yields } R_2''(z) = R_2' * e^{-A * s * \frac{z}{l}} \quad (6)$$

When we integrate the equation no. 6 through the whole slot diameter, we obtain the resulting current density, which depends on the factor  $A$ .

$$R_2'' = R_2' * \int_0^l e^{-A * s * \frac{z}{l}} dz = R_2' * \frac{A * s}{e^{A * s} - 1} \quad (7)$$

The Relation (7) is possible to simplify by exponential function, because both curves are almost identical.

$$R_2'' \approx R_2' * e^{-A * s} \quad (8)$$

By putting (8) into (2) we obtain the empirical relation (9)

$$\frac{M_i}{M_{i \max}} = \frac{2 * (1 + s_{zv} * e^{A * s})}{\frac{s}{s_{zv}} + \frac{s_{zv}}{s} + 2 * s_{zv} * e^{A * s}} \quad (9)$$

#### 4. Conditions of running and stability:

After comparison of the driving and braking torque we obtain the basic equation for steady state.

$$p * \frac{1 + s_{zv} * e^{A * s}}{\frac{s}{s_{zv}} + \frac{s_{zv}}{s} + 2 * s_{zv} * e^{A * s}} = (1 - s)^2, \text{ where } p = \frac{2 * M_{zv}}{M_b} * \left( \frac{U}{U_n} \right)^2 \quad (10,11)$$

A drive will be in stable state when the no-equation (11) is valid.

$$\left( \frac{\partial M_h}{\partial \omega} \right)_{\omega_0} < \left( \frac{\partial M_z}{\partial \omega} \right)_{\omega_0} \quad \text{or with slip} \quad \left( \frac{\partial M_h}{\partial s} \right)_{s_0} > \left( \frac{\partial M_z}{\partial s} \right)_{s_0} \quad (12)$$

After putting (9, 10) into (11) we get following equation.

$$(1-s) * \frac{\partial}{\partial s} \left( \frac{1 + s_{zv} * e^{A*s}}{\frac{s}{s_{zv}} + \frac{s_{zv}}{s} + 2 * s_{zv} * e^{A*s}} \right) > 2 * \frac{1 + s_{zv} * e^{A*s}}{\frac{s}{s_{zv}} + \frac{s_{zv}}{s} + 2 * s_{zv} * e^{A*s}} \quad (13)$$

The worst case is when  $A = 0$ . For the rest of cases is the unstable zone smaller. After calculation we obtain no-equation no. 14.

$$3 * s^3 + (4 * s_{zv}^2 - 1) * s^2 + s_{zv}^2 * s + s_{zv}^2 > 0 \quad (14)$$

If this relation valid, the drive is stable. If  $A > 0$  we obtain the transcendent relations, it is not possible to find analytical solution.

We can see from picture no. 4, that drive will be stable in whole speed range. But pull-out slip ( $s_{zv}$ ) of machine must be greater then 11%.

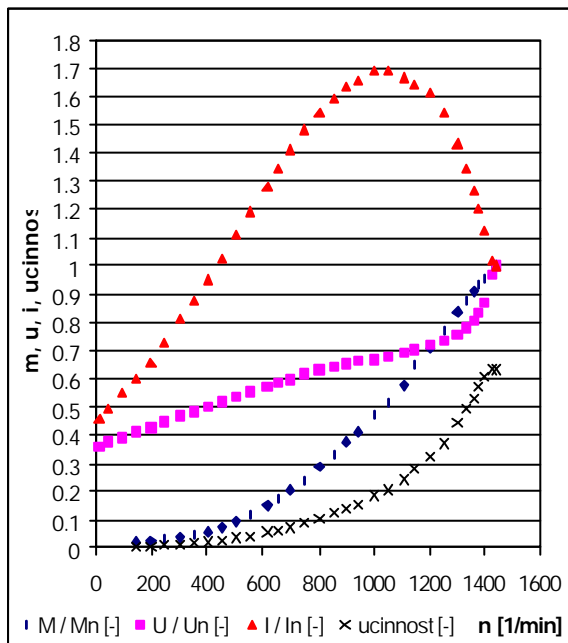


Figure 5. –  $M, U, I, \eta$  as functions of speed with quadratic load characteristic.

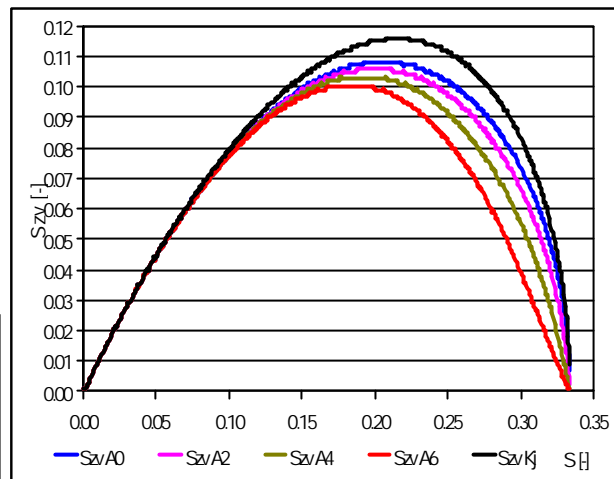


Figure 4. – Range of unstable drive, it is function of slip and pull-out slip

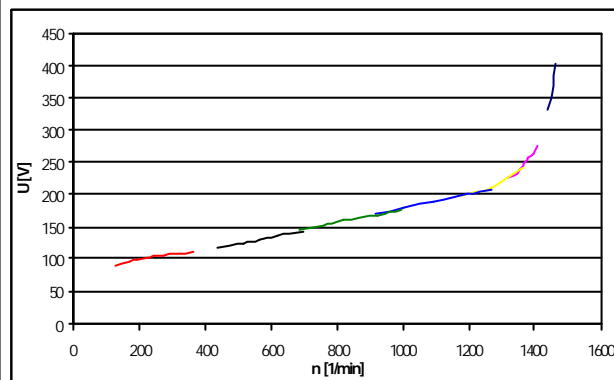


Figure 6. – Speed as a function of voltage  $\pm 10\%$  of nominal voltage - single curve for single voltage

## 5. Estimation:

From experiments and calculations we can say that is possible to change the speed of IM with quadratic torque characteristic within whole range. The unstable zone is only for machines with the pull-out torque smaller than 10.6%. Little machines have this slip about 15 - 20%. That is why it is possible to change the speed within whole range. But this kind of control has many disadvantages. The efficiency has quadratic decrease with speed, the current is greater then nominal one. The self-cooling at low speeds is very bad and machine is overheated. The big advantage is the use of power converter, for example AC/AC thyristor converter. This kind of regulation is used in laboratories, but the forced cooling is requested.